Math and Physics contest "Phystech School 2023"

Problem Set #2

recommended for grades 7 and 8

Math problems

1. (3 points) Puss in Boots has three days to complete a mission. His promised reward is 950 pesos. On the first day, he was paid 475 pesos, on the second day he was paid 2/5 of the amount paid on the first day and on the third day he will be given the rest of the reward. How many pesos will Puss receive on the third day?

2. (3 points) At a petroleum plant, in total 5 m³ of gasoline and kerosene were produced. The volume of kerosene is 25% greater than that of gasoline. Find the volumes kerosene and of gasoline?

3. (4 points) At the Sorochyntsi Fair, they sell a rhombus-shaped "Giant" crêpe with diagonals of 18 cm and 25 cm, as well as a round "Classic" crêpe whose circumference is $(0.3 \times \pi)$ m. Nataly ate the "Giant" crêpe, and Kate ate the "Classic" crêpe. Which girl ate more?

4. (6 points) Solve the following equation graphically:

$$\frac{8}{x} = \sqrt{x}.$$

5. (10 points) The Minions have forgotten the last digit of the code needed to launch their spaceship. In the instructions for the ship, it is written that the last digit of the code is the square of the following expression:

$$\frac{2}{4-x} + \left(\frac{3}{18}\right)^1 + \left(\sqrt{5}\right)^0 + \frac{4+x}{(-6x+9+x^2)} \div \frac{(-16+x^2)}{(-6+2x)} - \sqrt{1\frac{1}{36}} \cdot \sqrt{1\frac{12}{37}},$$

where $x = \frac{7}{3}$. What is this digit?

6. (11 points) Alex and Nick are participating in a competition, where together they have to answer 250 questions as quickly as possible. For some time t, Alex was answering the questions at a rate of 9 answers per minute. When he got tired, Nick took his place and answered the remaining questions. Nick was responding for the same period of time t. If each boy answered the questions at the same speed as in the competition, but Alex – during time t, and Nick – within 12.5 min, then Nick would have answered twice as many questions as Alex. Find t.

7. (12 points) Triangles ABC and BA_1C_1 are placed so that $\overline{AA_1}$ is a median of $\triangle ABC$ and a bisector of $\angle BA_1C_1$. $\overline{AA_1}$ is divided into three equal parts by the point of intersection of the medians of $\triangle ABC$ and by the point of intersection of the bisectors of $\triangle BA_1C_1$. The quadrilateral A_1AC_1C is a trapezoid. Find all the angles of $\triangle BA_1C_1$.

Physics problems

1. (3 points) Alex was the first one to finish a 30 km race. It took him 2 h 24 min. Find Alex's average speed in km/h.

2. What is the period of rotation of:

a) (2 points) the Earth around the Sun in the reference frame of the Sun?

b) (2 points) the Sun around the Earth in the reference frame of the Earth?

3. (4 points) To prepare coffee with milk, a barista pours 100 g of coffee at a temperature of 100°C and 100 g of milk at 5°C into a cup. The specific heat capacities of coffee and milk are equal. What will be the equilibrium temperature in the cup? Consider that while the equilibrium temperature is being reached, the coffee and milk are not exchanging heat with the environment.

4. (5 points) Once during a school celebration, Riley was watching balloons flying away. He thought: "What should be the size of a balloon for it to lift me above the ground?". Find the minimum volume of a helium-filled balloon that can lift Riley into air if Riley grabs it. The mass of the boy is 50 kg, the air density is 1.29 kg/m³. Ignore the masses of the balloon and of the helium inside it.

5. (10 points) During an aerial photography mission, a drone with failed communication equipment was captured three times at different locations. As long as there is fuel, the drone moves in the autopilot mode along a circular trajectory at a constant speed. At 06:10 AM the drone was at point 1 (see figure), at 06:19 AM it was at point 2, at 07:10 AM it was at point 3. Redraw the figure on squared paper and mark the point where the drone will be at 07:16 AM, if it doesn't run out of fuel. You can use a protractor.