

Math and Physics contest “Phystech School 2023”

Problem Set #3

recommended for grades 9-11(12) and adults

Mathematics problems

1. Student Victor got into a car with his parents in Kyiv and drove home to Kherson. The length of the route is 650 km, and the speed of the car is 130 km/h. At a certain moment of the trip, Victor asked his mother how much more to go. She replied that they had driven 60% of the way.

a) (1 point) How many kilometers had Victor traveled already?

b) (2 points) How much more time do they need to reach their destination?

2. (5 points) Borys got into sports and found a daily push-up program. On the first week Borys did 700 push-ups. On each following week Borys did 140 more push-ups than on the previous week. How many push-ups in total had Borys done in 12 weeks of the training?



3. (5 points) Volunteers buy food for cat shelters. In the first shelter 3 cats and 5 kittens eat 2,700 g of food per day. In the second shelter 6 cats and 9 kittens eat 5,040 g of food per day. Assume that in these shelters every day all the cats eat the same amount of food, as well as the kittens. How much food does a cat eat in a shelter per day, and how much food does a kitten eat per day? If the answer means that a kitten eats more than a cat, explain why this may happen.

4. (5 points) A round medal was produced from a workpiece shaped as a rhombus with a side of 10 cm. Its inscribed circle has a radius of 3 cm. Calculate the area of the workpiece.

5. (5 points) An engineer designs a pyramid, whose base is a rectangle with a diagonal of 24 cm. The height of the pyramid is 40 cm, all its lateral edges are equal. Find the length of a lateral edge.

6. Olivia studies radio waves. While working on a Wi-Fi router, she discovered that the frequency of its waves changes over time according to the formula

$F(t) = 5000 + 11 \sin\left(\frac{\pi t}{10}\right)$, where $F(t)$ is the signal frequency in megahertz, t is the time in seconds that elapsed since the beginning of the observation.



- a) (5 points) At which time t on the interval $25 \leq t \leq 35$ will the frequency equal to 5,000 megahertz?
- b) (2 points) Find the maximum and minimum values of frequency.

7. (10 points) A fire bucket has the shape of a right circular cone. Its slant height is 41 cm, and the sum of its height and radius is 49 cm. Find the volume of the bucket.



8. (12 points) Max has 7 nickels (5 cent coins), 9 dimes (10 cent coins), and 4 quarters (25 cent coins). His brother offers Max to play one of the two following games. The loser will buy an ice cream for the winner.



- 1) Take two coins alternately at random and win if the value of the second coin is greater than that of the first one.
- 2) Randomly take two coins and win if at least one of them is a quarter (25 cents).

In which of these games does Max have a greater chance of winning?

9. (15 points) A university has student self-governance, which helps the university to solve educational and everyday issues. The self-governance council consists of 20 students, who choose a chairman from 4 candidates once a year. Each of the twenty students votes for only one candidate. At the end of the election, the council submits a protocol where the numbers of votes for each candidate are recorded, but this protocol does not indicate who voted for whom. How many different variants of the protocol can there be?

Physics problems

1. (4 points) Michael dreams of becoming a professional cyclist. For a dream to come true, he often rides a bike along his favorite route Ternopil–Lviv, the length of which is 130 km. On Saturday morning Michael left Ternopil at a speed of 24 km/h and rode for 3 hours, then he had a rest in the shade for 1 hour, and then he covered the remaining distance in 2 hours. Find Michael's average speed.



2. (4 points) Doctors use the yttrium-90 isotope to treat certain diseases. The half-life of this isotope is 64 hours. How many times will the number of yttrium-90 atoms in a certain volume decrease in 8 days?

3. (5 points) A hummingbird weighing 5 g is standing on a table. The area of each foot of the bird is 0.25 cm^2 . How much pressure does the hummingbird exert on the table?

4. (8 points) Thorin Oakenshield ordered piedmont jewelers to make a golden spear weighing 15 kg. To do this, the dwarves need to melt 15 kg of gold stored at a temperature of 14°C . The jewelers have 3 magic balls, each capable of giving 1 MJ of thermal energy. Will they be enough to melt the gold? The specific heat capacity of gold is $130 \text{ J}/(\text{kg}\cdot^\circ\text{C})$, the heat of fusion of gold is $67 \text{ kJ}/\text{kg}$, the melting point of gold is $1,064^\circ\text{C}$.

5. At the Vernadsky Antarctic Research Base, a scientist examines a sample using a magnifying glass with an optical power of +5 diopters, placing it at a distance of 10 cm from the magnifying glass. Find:

- a) (1 point) the focal length of the lens;
- b) (4 points) how far from the magnifying glass the image of the sample is located;
- c) (2 points) if this image real or virtual;
- d) (4 points) the magnification of the magnifier.

6. There is 5kg of chlorine Cl_2 at a shoe factory. During isobaric expansion its temperature increased by 15°C .

- a) (6 points) What is the work done by the gas?
- b) (6 points) Did its internal energy increase or decrease? By how much?

The molar mass of the chlorine atom is $35.5 \text{ g}/\text{mol}$, the universal gas constant is $8.31 \text{ J}/(\text{mol}\cdot^\circ\text{C})$, and the specific heat capacity of chlorine at a constant pressure is $500 \text{ J}/(\text{kg}\cdot^\circ\text{C})$.

7. (18 points) The electric circuit shown in the figure below is composed of 23 different ammeters, 23 different resistors, and 23 similar voltmeters. All the ammeters and voltmeters give correct readings, but they may not be ideal. The first ammeter shows 8 mA, the second – 7.5 mA, the third – 7.2 mA. The second voltmeter shows 12 V. Find the sum of the readings of all the 23 voltmeters.

